Planar Topology

Math 87 (Spring 2023)

Ryan Maguire
Ryan.J.Maguire.GR@dartmouth.edu

Class Time: Th 1:05 - 3:15, Kemeny 121
Office Hours: MW 3:30 - 5:00, Kemeny 241

Course Description

Point-set topology covers things like topological and metric spaces, discussing topics like separation and countability properties, and perhaps manifolds if time permits. Algebraic topology deals with homology, cohomology, and homotopy theory. Differential topology extends the topological ideas of manifolds to a new setting in which one can conduct calculus and study things like curvature. There are several topics in between that have been somewhat forgotten but are used daily by working mathematicians, even if they do not know it.

In this course we'll talk about continua theory and planar topology, and to some extent the general study of topological manifolds. Many of the great theorems of topology such as the Jordan curve theorem, Schoenflies theorem, and Brouwer fixed-point theorems have classical proofs that explore different ideas than the homological proofs one finds in an algebraic topology textbook. We'll discuss these ideas and go through these proofs.

Course Objectives

By the end of the course one should have an understanding of continua theory, planar topology, topological manifolds, and homotopy groups. If there's time we may dive into some homological ideas.

Prerequesites

The only prerequisite is a course in point-set topology (Math 54). We'll briefly review these ideas before the main subject, but still one should have seen the material in-depth before hand.

Textbook

There is **no required textbook**. I will be handing out notes that will cover everything needed in the class. For those who become really fascinated with topology, the following are *recommended*, but not required.

- *General Topology* by Stephen Willard

- *General Topology* by John L. Kelley
- *Introduction to Topological Manifolds* by John Lee

Grading

This is a pass/fail course that is project based. Students will meet with me to discuss progress and at the end of the term they will present to the class. Depending on the topic chosen, either a poster or slides will suffice.

Order of Topics

This being somewhat of a topics course, the material is likely to change. Here's my current idea of the semester.

1. Review of Point-Set Topology
    - The Hausdorff property.
    - Continuity, homeomorphisms, homotopy, and homotopy equivalence.
    - Metric spaces and countability properties.
2. Topological Continuum
    - Definitions.
    - Examples (Lakes of Wada, Warsaw Circle, etc.)
    - Basic Properties.
3. Planar Topology
    - The Kuratowski Graph Theorem.
    - Kuratowski's Indecomposibility Theorem.
    - Jordan Curves and Jordan Arcs.
4. The Jordan Curve Theorem
    - Proof of the Theorem
    - The Schoenflies Theorem
    - The Jordan Schoenflies Theorem
5. More Planar Topology
    - The Denjoy-Riesz Theorem
    - Osgood Curves, Lebesgue Measure, and Julia Sets.
    - Janiszewski's Theorem.
6. Partial Results in Higher Dimensions

- The Jordan-Bouer Theorem (Generalized Jordan Theorem).
- The Alexander Horned Sphere.
- Antoine's Necklace
- The Brouwer Fixed Point Theorem

7. Topological Manifolds and Topological Groups
    - Basic definitions and properties.
    - Examples.
    - Applications of Topological Groups.
    - Lie Groups and Smooth Manifolds.
8. Homotopy Groups
    - The Fundamental Group.
    - Homotopy Groups of Spheres.
    - The Hopf Fibration.
9. Duality
    - Alexander Duality
    - Generalizations of the Jordan Curve Theorem.

Disabilities

Students with disabilities who may need disability-related academic adjustments and services for this course are encouraged to see me privately as early in the term as possible. Students requiring disability-related academic adjustments and services must consult the Student Accessibility Services office (Carson Hall, Suite 125, 646-9900). Once SAS has authorized services, students must show the originally signed SAS Services and Consent Form and/or a letter on SAS letterhead to their professor. As a first step, if students have questions about whether they qualify to receive academic adjustments and services, they should contact the SAS office. All inquiries and discussions will remain confidential.

Stress and Mental Well-Being

The academic environment at Dartmouth is challenging, our terms are intensive, and classes are not the only demanding part of your life. There are a number of resources available to you on campus to support your wellness, including your undergraduate dean, Counseling and Human Development, and the Student Wellness Center.

Religious Observances

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me before the end of the second week of the term to discuss appropriate accommodations.